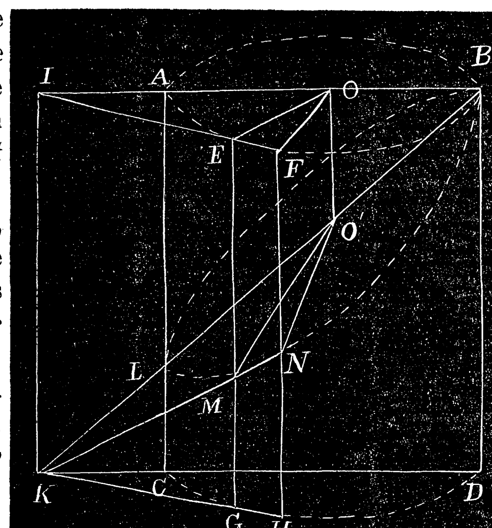
give the proofs that the rules of operation found in III. are applicable to the forms of combination here set forth, but here also I must refer to my extended treatise in which these proofs are drawn out with all requisite exactness; and where at the same time the development advances in such a way that every thing, which seems to be arbitrary in the presentation of the different ideas, vanishes.

GEOMETRICAL DETERMINATION OF THE SOLIDITY OF THE ELLIPSOID.

BY OCTAVIAN L. MATHIOT, BALTIMORE, MARYLND.

LET $ABCD$ be a right cylinder bounding a regular prism having an infinite number of sides, and let $EFGH$ represent one of these sides. Draw FEI tangent to the cylinder at F and meeting the diameter AB produced in I. From I let fall the perpendicular IK to meet the lower diameter CD produced in K. From K draw KGH a tangent to the cylind. at H.



From B to K pass a plane with cutting edge parallel to the lower base, and it will produce the ellipse $LMNB$ containing a polygon having MN for one of its infinite number of sides.

LB, the continuation of KL, will be the transverse axis of the ellipse, while the conjugate axis will be equal to the diameter AB of the cylinder.

From the centre O of the upper base let fall the perpendicular OO' and it will pass thro' the centre O' of the ellipse.

From O draw OE and OF, and from O', $O'M$ and $O'N$. From the similar triangles BOO' and BIK, we have $BO : BO' :: OI : O'K$. Since BO is half of AB and BO' is half of BL, $BO =$ semi-conjugate diameter $= B$, and $BO' =$ semi-transverse $= A$; $\therefore$ $OI : OK :: B : A$ But OI is the subtangent corresponding to the tangent line FEI of the circle, while $O'K$ is the subtangent crresponding to the tangent line KMN of the ellipse.

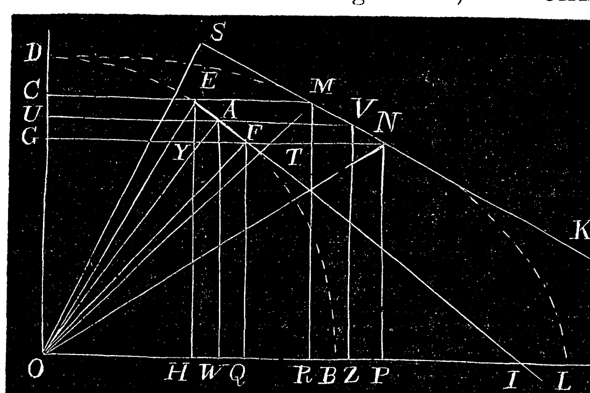
From the construction of the solid it will be seen that the triangle $O'MN$ is directly beneath the triangle OEF, and every triangle that can be drawn to the sides of the polygon in the circle will have its correlative triangle directly beneath it drawn to the sides of the polygon formed in the ellipse, and it can easily be shown that, for all such correlative triangles, the subtangent of the triangle in the circle is to the subtangent of the triangle in the ellipse as B to A.

It is also evident from the construction that any line drawn from the circumference of the circle $AEFB$ perpendicular to the diameter AB must be equal in length to the line that is directly beneath it drawn from the circumference of the ellipse perpendicular to the the transverse axis BL.

Lines dropt from the vertices E and F perpendicularly on the diameter AB must be equal to lines dropt from vertices M and N perpdndicularly on the transverse axis BL; and these equal perpendiculars indicate the corresponding vertices of the correlative triangles.

Let $DMNL$ represent the ellipse, OL one half the transverse axis, and OD one half the conjugate axis.

With OD as a radius inscribe the circle $DEFB$, and let EF represent one of the infinite number of sides of the inscribed regular polygon. Draw OE and through E and F the vertices of the triangle EFO, draw CEM and GFN parallel to OL, and intersecting the ellipse at M and N. From E and F let fall on OL the perpendiculars EH and FQ, also from M and N let fall on OL the perpendicul's MR and NP. Then, becanse they lie between parallel lines, EH and FQ in the circle are respectively equal to MR and NP in the ellipse; $\therefore$ M and N are the vertices of the triangle that is correlative to OEF.



Join OM and ON; the triangles OMN and EFO being correlative their subtangents reckoning from O are to each other as A to B.

Draw MNK a tangent to the ellipse at M intersecting OL produced in K, and draw EFI tangent to the circle at I and intersecting OB produced in I; then OI is to OK as B to A.

Prolong the tangent KNM to meet OS drawn perpendicular to it,

then from the similar triangles NMT and KOS we have the proportion

$$MT : SO :: MN : OK;$$

$\therefore MT \times OK = SO \times MN$. But $SO \times MN$ equals double the area OMN;
$\therefore MT \times OK =$ equals double the area OMN.

From A the middle of EF let fall the perpendicular AO. Then from the similar triangles EYF and OAI we have

$$EP : AO :: EF : OI;$$

$\therefore EY \times OI = AO \times EF$. But $A\ O \times\ EF$ equals double the area EFO;
$\therefore EY \times OI =$ equals double the area EFO.

Therefore $MNO : EFO :: MT \times KO : EY \times IO :: A : B$ (because MT and EY are between parallels and are therefore equal, and OK is to OI as A to B); $\therefore$ area $OMN = (A \div B) \times$ area OEF.

Through the middle of EF draw UAV paralell to OK and bisecting MN in V, and from A and V let fall the equal perpendiculars, AW and VZ, on the axis OK.

The solid generated by revolving the triangle OMN around the axis OK is represented by area $OMN \times \frac{4}{3}\pi . VZ$. Or, since $MNO = (A \div B) \times OEF$, and $OEF = EF \times \frac{1}{2} AO$, the solid is $\frac{2}{3}\pi . EF.AW.AO \times (A \div B)$.

From the similar triangles EYF and AWO we find $EF \times AW = HQ \times AO$; and by substituting this equivalent for $EF \times AW$ in the above expression we have $\frac{2}{3}\pi . AO^2 . HQ \times (A \div B)$. When the number of sides of the inscribed polygon is infinite, $AO = OD = B$, and hence the above expression for the solid generated by revolving the triangle MNO about the axis OK, is $\frac{2}{3}\pi . A . B \times HQ$. As this expression is true for all values of HQ it is true for their sum; hence, substituting for the several values of HQ their sum, $OB = B$, we get, for the solidity ot half the prolate ellipsoid $\frac{2}{3}\pi A . B^2$. Hence the solidity of the prolate ellipsoid is $\frac{4}{3}\pi . A . B^2$.

In a manner precisely similar to the foregoing it may be shown that the solidlty of the oblate ellipsoid is $\frac{4}{3}\pi . A^2 . B$.

GENERAL SOLUTION OF THE PROBLEM OF ANY NUMBER OF BODIES.

BY R. J. ADCOCK, ROSEVILLE, ILL.

THE masses of any number of bodies being given, together with their positions and motions at any given time with reference to any three rectangular axes, to find their positions and motions at any other time.